

# Interacting Higher Spins and the High Energy Limit of the Bosonic String

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## Abstract

In this note, we construct a BRST invariant cubic vertex for massless fields of arbitrary mixed symmetry in flat space-time. The construction is based on the vertex given in bosonic Open String Field Theory. The algebra of gauge transformations is closed without any additional, higher than cubic, couplings due to the presence of an infinite tower of massless fields. We briefly discuss the generalization of this result to a curved space-time and other possible implications.

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## 1 Introduction

While the low energy limit of string theory is fairly well understood the properties of its high energy limit [1]– [2] (see also [3]) still impose a series of questions. Several proposals and conjectures have been made about the high energy limit of String Theory and its role in the still mysterious M–theory.

For example, unlike various types of supergravities which describe the low energy approximation of superstring theories, the field theory description of the high energy regime of string theory is still unknown. It is also not known which kind of gravitational background is required for its consistency. Obviously, when dealing with very small distances, the only type of nontrivial background a string still might "feel" is a highly curved background.

Several results based on AdS/CFT duality strongly support the idea that there might be a relation between the high energy limit of String theory and Higher Spin (HS) gauge theories [4]. These and other considerations suggest that a possible candidate (if any) for a field theoretical description of String theory at high energies might be the HS gauge theory developed in [5]. This theory is defined on an AdS background in any space-time dimension and classically is a perfectly consistent theory. One can wonder if there is any precise connection between HS theory and string theory. For this reason it seems interesting to formulate HS field theory in a language similar to that used in String Theory like for example bosonic String Field Theory (SFT) [6, 7, 8]. Moreover one can try to find a generalization of the methods used in SFT to the case of curved backgrounds with constant curvature, such as AdS space-times [9]–[10].

A very useful method for describing HS theories is the BRST method developed for example in [9, 11, 12, 13] and the free theory formulation of HS theories based on the BRST approach has been explored to some extent. The problem of constructing interaction vertices at the high energy limit of String theory was considered previously by various authors [14, 15] or alternatively one can address the problem of an

interaction between higher spin fields without any recourse to string theory [5, 16].

Since the BRST description for various massive and massless fields belonging to *irreducible* representations of Poincare and AdS groups is available (see for example [9, 11, 12, 13]), one can try to find (self)interacting Lagrangians for them. Here though, we present another SFT inspired solution which proves to be exact to all orders of the coupling constant. The interaction we consider couples an infinite tower of *reducible* representations of the Poincare group, rather than irreducible ones. The reason behind this is that the problem of finding an interaction between reducible representations shares a lot of similarity with String Theory and this similarity might prove to be very helpful [7]. Moreover finding a formal analogy with string theory one can hope to understand a "physical" connection between these theories.

The cubic interaction discussed in the present paper is obtained from a formal high energy limit of conventional bosonic Open String Field Theory (OSFT) [17, 18, 19]. One can actually consider it as an OSFT counterpart of the high energy limit of string interactions obtained in [1, 2]. In [1, 2] the high energy limit of string theory is considered in the framework of perturbative string theory and therefore all string "in" and "out" states are essentially "on shell". The vertex we consider describes off-shell interactions of HS fields as one could expect from analogy with OSFT. In addition, a remarkable property of our vertex is that it does not necessarily require the presence of all kinds of mixed symmetry fields for its classical consistency, in contrast to OSFT. So one can consider a truncation of the theory to totally symmetric fields.

In sections 3 and 4 we show how to construct a cubic vertex for both totally symmetric fields and those with mixed symmetry. From this vertex we construct the associated nonabelian deformations of the gauge transformations. We demonstrate how the algebra of gauge transformations closes to all orders of the coupling constant  $g$ . Further on we show that the action is fully gauge invariant under the nonabelian gauge transformations, which implies that our vertex is exact. This is analogous to OSFT.

However, one very important question is to consider a supersymmetric extension of our construction and in particular for an HS theory on an *AdS* background. In this way one can try to make a connection with the high energy limit of string theory on a highly curved Anti de Sitter background, which could be relevant to AdS/CFT. However, a BRST description of supersymmetric HS theories on AdS is yet not fully understood. As a simpler problem we consider the deformation of the proposed bosonic vertex for flat background to the case of  $\mathcal{D}$  - dimensional AdS. The direct deformation of the vertex does not seem to be consistent. One can hope that considering a supersymmetric version of the system will give a solution result. We leave this for a future investigation.

In any case one can assume that the solution obtained in this paper might be a step (or at least a nontrivial toy model) towards a better understanding of the connection between interacting higher spin gauge theories and string theories.

## 2 Basic equations

A formal way to construct the nilpotent BRST charge at the high energy limit is to start with the BRST charge for the open bosonic string

$$\mathcal{Q} = \sum_{k,l=-\infty}^{+\infty} (C_{-k}L_k - \frac{1}{2}(k-l) : C_{-k}C_{-l}B_{k+l} :) - C_0, \quad (2.1)$$

perform the rescaling of oscillator variables

$$c_k = \sqrt{2\alpha'}C_k, \quad b_k = \frac{1}{\sqrt{2\alpha'}}B_k, \quad c_0 = \alpha'C_0, \quad b_0 = \frac{1}{\alpha'}B_0, \quad (2.2)$$

$$\alpha_k^\mu \rightarrow \sqrt{k}\alpha_k^\mu$$

and then take  $\alpha' \rightarrow \infty$ . In this way one obtains a BRST charge

$$Q = c_0l_0 + \tilde{Q} - b_0\mathcal{M} \quad (2.3)$$

$$\tilde{Q} = \sum_{k=1}^{\infty} (c_k l_k^+ + c_k^+ l_k), \quad \mathcal{M} = \sum_{k=1}^{\infty} c_k^+ c_k, \quad l_0 = p^\mu p_\mu, \quad l_k^+ = p^\mu \alpha_{k\mu}^+ \quad (2.4)$$

which is nilpotent in any space-time dimension. The oscillator variables obey the usual (anti)commutator relations

$$[\alpha_\mu^k, \alpha_\nu^{l,+}] = \delta^{kl}\eta_{\mu\nu}, \quad \{c^{k,+}, b^l\} = \{c^k, b^{l,+}\} = \{c_0^k, b_0^l\} = \delta^{kl}, \quad (2.5)$$

and the vacuum in the Hilbert space is defined as

$$\alpha_k^\mu |0\rangle = 0, \quad c_k |0\rangle = 0 \quad k > 0, \quad b_k |0\rangle = 0 \quad k \geq 0. \quad (2.6)$$

Let us note that one can take the value of  $k$  to be any fixed number without affecting the nilpotency of the BRST charge (2.3). Fixing the value  $k = 1$  one obtains the description of totally symmetric massless higher spin fields, with spins  $s, s-2, \dots, 1/0$ . The string functional (named "triplet" [6]) in this simplest case has the form

$$|\Phi\rangle = |\phi_1\rangle + c_0|\phi_2\rangle = |\varphi\rangle + c^+ b^+ |d\rangle + c_0 b^+ |c\rangle$$

whereas for an arbitrary value of  $k$  one has the so called "generalized triplet"

$$|\Phi\rangle = \frac{c_{k_1}^+ \dots c_{k_p}^+ b_{l_1}^+ \dots b_{l_p}^+}{(p!)^2} |D_{k_1, \dots, l_p}^{l_1, \dots, l_p}\rangle + \frac{c_0 c_{k_1}^+ \dots c_{k_{p-1}}^+ b_{l_1}^+ \dots b_{l_p}^+}{(p-1)!p!} |C_{k_1, \dots, k_{p-1}}^{l_1, \dots, l_p}\rangle,$$

where the vectors  $|D_{l_1, \dots, l_p}^{k_1, \dots, k_p}\rangle$  and  $|C_{l_1, \dots, l_p}^{k_1, \dots, k_p}\rangle$  are expanded only in terms of oscillators  $\alpha_k^{\mu+}$ , and the first term in the ghost expansion of (2.7) with  $p = 0$  corresponds to the state  $|\varphi\rangle$  in (2.7). One can show that the whole spectrum of the open bosonic string

decomposes into an infinite number of generalized triplets, each of them describing a finite number of fields with mixed symmetries [7].

In order to describe the cubic interactions one introduces three copies ( $i = 1, 2, 3$ ) of the Hilbert space defined above, as in bosonic OSFT [18]. Then the Lagrangian has the form

$$L = \sum_{i=1}^3 \int dc_0^i \langle \Phi_i | Q_i | \Phi_i \rangle + g \left( \int dc_0^1 dc_0^2 dc_0^3 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | | V \rangle + h.c. \right), \quad (2.7)$$

where  $|V\rangle$  is the cubic vertex and  $g$  is a string coupling constant. The Lagrangian (2.7) is completely invariant with respect to the nonabelian gauge transformations

$$\delta |\Phi_i\rangle = Q_i |\Lambda_i\rangle - g \int dc_0^{i+1} dc_0^{i+2} [(\langle \Phi_{i+1} | \langle \Lambda_{i+2} | + \langle \Phi_{i+2} | \langle \Lambda_{i+1} |) | V \rangle], \quad (2.8)$$

provided that the vertex  $|V\rangle$  satisfies the BRST invariance condition

$$\sum_i Q_i |V\rangle = 0. \quad (2.9)$$

The additional constraints imposed by the closure of the algebra of gauge transformations will be discussed in section 3.2. The gauge parameter  $|\Lambda\rangle$  in each individual Hilbert space has the ghost structure

$$|\Lambda\rangle = b^+ |\lambda\rangle \quad (2.10)$$

for the totally symmetric case, while the gauge parameters for the generalized triplets take the form

$$|\Lambda\rangle = \frac{c_{k_1}^+ \dots c_{k_p}^+ b_{l_1}^+ \dots b_{l_{p+1}}^+}{(p!)(p+1)!} |\Lambda_{k_1, \dots, k_p}^{l_1, \dots, l_{p+1}}\rangle + \frac{c_0 c_{k_1}^+ \dots c_{k_{p-1}}^+ b_{l_1}^+ \dots c_{l_{p+1}}^+}{(p-1)!(p+1)!} |\hat{\Lambda}_{k_1, \dots, k_{p-1}}^{l_1, \dots, l_{p+1}}\rangle.$$

Further on, in order to simplify equations in the following sections we introduce bilinear combinations of the oscillators

$$\gamma_{(kp)}^{+,ij} = c_{(k)}^{+,i} b_{(p)}^{+,j}, \quad \beta_{(kp)}^{+,ij} = c_{(k)}^{+,i} b_{0,(p)}^j, \quad M_{(kp)}^{+,ij} = \frac{1}{2} \alpha_{(k)}^{+, \mu, i} \alpha_{\mu(p)}^{+,j} \quad (2.11)$$

which have ghost number zero.

Let us make some comments about the BRST charge (2.3). We can actually justify the way it was obtained from the BRST charge of the open bosonic string since its cohomologies correctly describe equations of motion for massless bosonic fields belonging to mixed symmetry representations of the Poincare group (see e.g. [7]). So taking the point of view that, in the high energy limit the whole spectrum of the bosonic string collapses to zero mass, which is now infinitely degenerate, one can take the BRST charge (2.3) as the one which correctly describes this spectrum.

### 3 An exact cubic vertex for totally symmetric fields

#### 3.1 BRST invariance

We begin first with the simple case of a vertex for totally symmetric fields. This means we consider only one set of oscillators as in (2.5).

The form of the vertex can be deduced from the high energy limit of the corresponding vertex of OSFT. In bosonic OSFT the cubic vertex has the form

$$\begin{aligned}
 |V_3\rangle &= \int dp_1 dp_2 dp_3 (2\pi)^d \delta^d(p_1 + p_2 + p_3) \\
 &\times \exp \left( \frac{1}{2} \sum_{i,j=1}^3 \sum_{n,m=0}^{\infty} \alpha_{n,\mu}^{+,i} N_{nm}^{ij} \alpha_{m,\nu}^{+,j} \eta^{\mu\nu} + \sum_{i,j=1}^3 \sum_{n \geq 1, m \geq 0} c_n^{+,i} X_{nm}^{ij} b_m^{+,j} \right) |-\rangle_{123}, \\
 |-\rangle_{123} &= c_0^1 c_0^2 c_0^3 |0\rangle
 \end{aligned} \tag{3.1}$$

where the solution is given in terms of the Neumann coefficients and all string modes contribute. The oscillators  $\alpha_{0,\mu}^i$  are proportional to the momenta  $p_\mu^i$ . The vertex is invariant under the action of the BRST charge (2.1). In addition, the action (2.7) with the vertex (3.1) is invariant under the gauge transformations (2.8) to all orders in  $g$ .

As was mentioned in section 2 at the high energy limit the BRST charge takes the form (2.3) and can be truncated to contain any finite number of oscillator variables [7]. For this reason it is possible to look for the BRST invariant vertex that describes the interaction among only totally symmetric tensor fields of arbitrary rank, without the inclusion of modes with mixed symmetries. One possibility is to start from the SFT vertex (3.1) and keep in the exponential only terms proportional to at least one momentum  $p_\mu^r$ , therefore dropping all trace operators  $(\alpha_\mu^r \eta^{\mu\nu} \alpha_\nu^s)$ , as one does when obtaining the BRST charge (2.3) from (2.1) since they are leading in the  $\alpha' \rightarrow \infty$  limit. However, since these terms are exponentiated and the term  $\alpha_{n,\mu}^{+,r} N_{n0}^{rs} p_\mu^s$  is of the same order as  $\alpha_{n,\mu}^{+,r} N_{n0}^{rs} p_\mu^s$  ( $\alpha_{n,\mu}^{+,r} N_{nm}^{rs} \alpha_{m,\nu}^{+,s}$ ),  $m, n \geq 1$ , a priori one can keep them both. The same is true regarding the ghost part where although the term  $c_n^{+,r} b_0^s$  is leading compared to the term  $c_n^{+,r} X_{nm}^{rs} b_m^s$ ,  $n, m \geq 1$  one can not neglect the later one in the exponential. Let us stress that all these terms will be essential to maintain the off shell closure of the algebra of gauge transformations and complete gauge invariance of the action.

Based on the discussion above one can take the following ansatz for the vertex which describes interactions between massless totally symmetric fields with an arbitrary spin

$$|V\rangle = V^1 \times V^{mod} |-\rangle_{123} \tag{3.2}$$

where the vertex contains two parts: a part considered in [14]

$$V^1 = \exp ( Y_{ij} l^{+,ij} + Z_{ij} \beta^{+,ij} ). \tag{3.3}$$

and the part which ensures the closure of the nonabelian algebra

$$V^{mod} = \exp ( S_{ij} \gamma^{+,ij} + P_{ij} M^{+,ij} ), \quad (3.4)$$

where  $P_{ij} = P_{ji}$ . Putting this ansatz into the BRST invariance condition one can see that each part of the vertex should be invariant separately, namely one obtains

$$\tilde{Q} V^1 |-\rangle_{123} = \sum_i c^{+,i} (Y_{is} l_0^{is} - Z_{is} l_0^{ss}) V^1 |-\rangle_{123} = 0 \quad (3.5)$$

$$\begin{aligned} \tilde{Q} V^{mod} |-\rangle_{123} = \sum_i \{ - c^{+,i} \left( \frac{1}{2} (\delta^{ik} l^{+,li} + \delta^{il} l^{+,ki}) P_{kl} - S^{ik} l^{+,kk} \right) - \\ \beta^{+,ii} c^{+,m} S_{mi} \} V^{mod} |-\rangle_{123} = 0 \end{aligned} \quad (3.6)$$

Using momentum conservation  $p_\mu^1 + p_\mu^2 + p_\mu^3 = 0$  one can obtain a solution for  $Y^{rs}$  and  $Z^{rs}$

$$\begin{aligned} Z_{i,i+1} + Z_{i,i+2} &= 0 \\ Y_{i,i+1} &= Y_{ii} - Z_{ii} - 1/2 (Z_{i,i+1} - Z_{i,i+2}) \\ Y_{i,i+2} &= Y_{ii} - Z_{ii} + 1/2 (Z_{i,i+1} - Z_{i,i+2}). \end{aligned} \quad (3.7)$$

The first term in (3.6) should vanish on its own right. The terms proportional to  $c^{+,1}$  are

$$c^{+,1} [P_{12} l^{+,21} + P_{13} l^{+,31} + P_{11} l^{+,11} - S_{12} l^{+,22} - S_{13} l^{+,33} - S_{11} l^{+,11}] |-\rangle_{123} = 0 \quad (3.8)$$

with similar terms proportional to  $c^{+,2}$  and  $c^{+,3}$ . Using momentum conservation to eliminate  $p_\mu^3$  we arrive to the following solution:

$$\begin{aligned} S_{ij} = P_{ij} &= 0 \quad i \neq j \\ P_{ii} - S_{ii} &= 0 \quad i = 1, 2, 3 \end{aligned} \quad (3.9)$$

It actually turns out that all the other equations resulting from the ghost expansion of (3.6) lead to the same solution. The second term in (3.6) is trivially zero since the off diagonal components of  $S^{ij}$   $i \neq j$  vanish. So it does not result in any additional constraints, but this is not going to be the case for the mixed symmetry tensor fields considered in the next section.

### 3.2 The gauge transformations and closure of the gauge algebra

Having determined the form of the vertex from (3.7) and (3.9) we will proceed in computing the commutator of two gauge transformations with gauge parameters  $|\Xi\rangle$  and  $|\Lambda\rangle$ . In general, closure of the algebra to order  $O(g)$  implies

$$[\delta_\Lambda, \delta_\Xi] |\Phi_1\rangle = \delta_\Lambda |\Phi_1\rangle = Q_1 |\tilde{\Lambda}_1\rangle - g [(\langle \Phi_2 | \langle \tilde{\Lambda}_3 | + \langle \Phi_3 | \langle \tilde{\Lambda}_2 |) | V \rangle] + O(g^2) \quad (3.10)$$

where

$$|\tilde{\Lambda}_1\rangle = g(\langle\Lambda_2|\langle\Xi_3| + \langle\Lambda_3|\langle\Xi_2|)|V\rangle + O(g^2). \quad (3.11)$$

One can check that the closure of the algebra at the first order in  $g$  is equivalent to the BRST invariance of the vertex [18] which is satisfied by construction. To ensure the invariance in higher orders in  $g$  one might have to consider quartic and higher order vertices. Obviously it is not guaranteed in general that such a procedure will lead to the closure of the algebra. It should be emphasized that unlike the case of free triplets where the total Lagrangian splits into an infinite sum of individual ones [7] for the case of interacting triplets the fields  $|\Phi_i\rangle$  of (2.7) need to be an infinite tower of HS triplet fields, at least when the vertex is defined via (3.2) or (3.3). In other words:

$$|\Phi_i\rangle \rightarrow \sum_{s=0}^{\infty} |\Phi_i^{(s)}\rangle \quad (3.12)$$

In what follows we will assume cyclic symmetry in the three Hilbert spaces which implies along with (3.7)

$$\begin{aligned} Z_{12} &= Z_{23} = Z_{31} = Z_a \\ Z_{21} &= Z_{13} = Z_{32} = Z_b = -Z_a \\ Y_{12} &= Y_{23} = Y_{31} = Y_a \\ Y_{21} &= Y_{13} = Y_{32} = Y_b \\ Y_{ii} &= Y, \quad Z_{ii} = Z, \quad P_{ii} = -S_{ii} = -S = P \end{aligned} \quad (3.13)$$

Finally, it is instructive to give the gauge transformations based on (2.8)

$$\begin{aligned} \delta(|\varphi_1\rangle + c_1^+ b_1^+ |d_1\rangle + c_1^0 b_1^+ |c_1\rangle) &= (l_{11}^+ + l_{11} c_1^+ b_1^+ + c_1^0 b_1^+ l_0^1) |\lambda_1\rangle + \\ &+ g e^{(S c_1^+ b_1^+)} \{ -Z_a (\langle\varphi_2| \langle\lambda_3| |A\rangle - S (\langle d_2| \langle\lambda_3| |A\rangle)) + \\ &+ (Z Z_a - Z_b^2) \langle c_2| \langle\lambda_3| |A\rangle + (2 \leftrightarrow 3, Z_a \rightarrow Z_b) \} \end{aligned} \quad (3.14)$$

where for convenience we have defined the matter part of the vertex

$$|A\rangle = \exp ( Y_{ij} l^{+,ij} + P \sum_{i=1}^3 M^{+,ii} ) |0\rangle. \quad (3.15)$$

In what follows we will show that the vertex defined in (3.2) allows us to close the algebra of gauge transformations (3.10) without any order  $O(g^2)$  modifications. Let us assume that there are no quadratic and higher order terms in  $g$  in the gauge transformation law (3.10). The commutator of two gauge transformations is

$$\begin{aligned} [\delta_\Lambda, \delta_\Xi] |\Phi_1\rangle &= Q_1 |\tilde{\Lambda}_1\rangle \\ &+ g^2 [ \langle V | (|\Phi_1\rangle |\Lambda_3\rangle + |\Lambda_1\rangle |\Phi_3\rangle) \langle \Xi_3 | |V\rangle + \langle V | (|\Phi_1\rangle |\Lambda_2\rangle + |\Lambda_1\rangle |\Phi_2\rangle) \langle \Xi_2 | |V\rangle \\ &- \langle V | (|\Phi_1\rangle |\Xi_3\rangle + |\Xi_1\rangle |\Phi_3\rangle) \langle \Lambda_3 | |V\rangle - \langle V | (|\Phi_1\rangle |\Xi_2\rangle + |\Xi_1\rangle |\Phi_2\rangle) \langle \Lambda_2 | |V\rangle \end{aligned} \quad (3.16)$$



where we have suppressed the integrations over the ghost fields of (2.8). Evaluating the LHS of (3.16) using (2.8) and plugging it into the RHS of the expression for  $|\tilde{\Lambda}\rangle$  from (3.11) we obtain

$$\begin{aligned} & \langle V | (|\Phi_1\rangle|\Lambda_3\rangle + |\Lambda_1\rangle|\Phi_3\rangle) \langle \Xi_3 || V \rangle + \langle V | (|\Phi_1\rangle|\Lambda_2\rangle + |\Lambda_1\rangle|\Phi_2\rangle) \langle \Xi_2 || V \rangle \\ & - \langle V | (|\Phi_1\rangle|\Xi_3\rangle + |\Xi_1\rangle|\Phi_3\rangle) \langle \Lambda_3 || V \rangle - \langle V | (|\Phi_1\rangle|\Xi_2\rangle + |\Xi_1\rangle|\Phi_2\rangle) \langle \Lambda_2 || V \rangle \\ & = \langle V | (|\Xi_1\rangle|\Lambda_2\rangle + |\Lambda_1\rangle|\Xi_2\rangle) \langle \Phi_2 || V \rangle + \langle V | (|\Xi_1\rangle|\Lambda_3\rangle + |\Lambda_1\rangle|\Xi_3\rangle) \langle \Phi_3 || V \rangle \end{aligned} \quad (3.17)$$

Equation (3.17) is valid for any value of the vertex, but using the solution (3.2), (3.7), (3.9) one can see that the expression in (3.11) vanishes identically. A typical term to demonstrate this is:

$$\begin{aligned} \langle \tilde{\Lambda}_3 | \sim \langle V || \Xi_1 \rangle | \Lambda_2 \rangle &= \int dc_0^1 dc_0^2 {}_{123} \langle - | \exp(-Z_{ij} c^i b_0^j - S^* c^i b^i) \\ &\times b_1^+ b_2^+ \langle \langle A || \xi_1 \rangle | \lambda_2 \rangle | 0_1 \rangle_{gh} | 0_2 \rangle_{gh}. \end{aligned} \quad (3.18)$$

It is impossible to make the RHS of the expression above proportional to  $b^3$  as is necessary from (2.10) since this requires the vertex to have terms like  $c^2 b^3$  and  $c^1 b^3$  which are absent since  $S_{ij}$  is diagonal. Now this implies that the RHS of (3.17) vanishes identically. Therefore the LHS of (3.17) will have to vanish. Actually it will turn out that each term of the LHS of (3.17) vanishes identically. Since the computation is long we present only a representative term:

$$\begin{aligned} \langle V || \Phi_1 \rangle | \Lambda_3 \rangle \langle \Xi_3 || V \rangle &= \int dc_0^2 dc_0^3 \left( \int dc_0^3 dc_0^1 {}_{123} \langle - | V^{\dagger, gh} \langle A^\dagger | \times \right. \\ & (|\varphi_1\rangle + c_1^+ b_1^+ |d_1\rangle + c_0^1 b_1^+ |c_1\rangle) b_3^+ |\lambda_3\rangle) \times \\ & \times \langle \xi_3 | b_3 | A \rangle V^{gh} | - \rangle_{123} \end{aligned} \quad (3.19)$$

where we have denoted

$$V^{gh} = \exp(Z_{ij} c^i b_0^j + S c^i b^i) \quad (3.20)$$

The integration in the parentheses can be easily performed and it gives us a state in Hilbert space number two

$$- {}_{gh} \langle 0_2 | (Z_b (\langle A^\dagger | \varphi_1 \rangle - S \langle A^\dagger | d_1 \rangle) - (Z Z_b - Z_a^2) \langle A^\dagger | c_1 \rangle) |\lambda_3\rangle \exp(-S^* c^2 b^2) \quad (3.21)$$

Inserting the expression above in (3.19) and performing the second integration of ghosts we get

$$\begin{aligned} & Z_a \{ Z_b (\langle A^\dagger | \varphi_1 \rangle - S \langle A^\dagger | d_1 \rangle) - (Z Z_b - Z_a^2) \langle A^\dagger | c_1 \rangle \} |\lambda_3\rangle \times \\ & \times (1 - |S|^2) \langle \xi_3 | A \rangle \exp(S c_1^+ b_1^+) | 0_1 \rangle_{gh} \end{aligned} \quad (3.22)$$

where the factor  $(1 - |S|^2)$  came from saturating the second Hilbert space ghost vacuum:

$$\int dc_0^2{}_{gh} \langle 0_2 | \exp(-S c^2 b^2) V^{gh} | - \rangle_{123} = \quad (3.23)$$

$$-(1 - S^2) \exp(Z_{ij} c^i b_0^j + S c^i b^i) |_{i,j \neq 2} c_0^1 c_0^3 | 0_1 \rangle_{gh} | 0_3 \rangle_{gh}.$$

Proceeding in the same manner we can compute all terms and collect them in (3.17). The expressions proportional to  $|\varphi\rangle$  and  $|d\rangle$  in (3.22) cancel among each other without any further constraints on the parameters. On the other hand, cancelation of terms proportional to  $|c\rangle$  in (3.22) is non trivial. One can check a few examples which involve lower numbers of derivatives in the interaction part of the Lagrangian.

Checking the invariance with respect to the gauge transformations which involve the gauge parameters  $\lambda$ ,  $\xi$  and the scalar field  $c$  one can suppose the existence of a solution  $Y = Z = 0$  and  $Y_a = -Z_a$ ,  $Y_b = Z_a$  with an arbitrary value of  $S$ . However, this solution does not allow one to close the algebra when the parameters of gauge transformations of a higher rank tensor states  $c_{\mu_1 \dots \mu_n}$  are involved. Therefore (3.22) imposes <sup>‡</sup>

$$|S|^2 = 1. \quad (3.24)$$

Actually (3.24) implies that each term of (3.17) should vanish separately. This leads to a trivial commutator:

$$[\delta_\Lambda, \delta_\Xi] |\Phi_1\rangle = 0 \quad (3.25)$$

or rather to

$$\delta_\Lambda \delta_\Xi |\Phi_1\rangle = 0. \quad (3.26)$$

In other words we can consider the vertex (3.2) as a field dependent deformation of the BRST charge in (2.3), which can be written schematically

$$Q' = Q + gV(\Phi) \quad (3.27)$$

with the nilpotency property

$$Q'^2 = Q^2 + 2gQV(\Phi) + g^2V(\Phi)^2 = 0 \quad (3.28)$$

which follows from the nilpotency of  $Q$ , the BRST invariance of the vertex (2.9) and (3.24). Proceeding further with analogy with the String Field Theory one can make both the string functional and gauge transformation parameters to be matrix valued (i.e., introduce Chan–Paton factors). The resulting theory will still satisfy (3.26).

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<sup>‡</sup> Note that there is a potential infinity coming from the matter part of (3.22). Indeed there are terms  $\langle 0 | e^{(PM)} e^{(PM^+)} | 0 \rangle$ . Depending on the dimensionality of the space-time  $\mathcal{D}$  it can have a logarithmic divergence or higher. This might seem to invalidate our condition (3.24), but nevertheless, the condition (3.24) makes the expression in (3.22) strictly zero. So in the spirit of (3.30) we have an infinite series of contributions which cancel term by term. We believe that this is the correct prescription in dealing with this issue.

Now we should check whether the cubic vertex we constructed is exact. In other words the action (2.7) should be invariant under (2.8) to all orders in  $g$ . We should point out that closure of the algebra does not ensure this. The simplest counter example is the  $U(1)$  scalar electrodynamics. The gauge transformation of the scalar,  $\delta\varphi \sim i\lambda\varphi$  is derived from (2.8) and the form of the cubic interaction of scalar electrodynamics. The algebra of gauge transformations is abelian as in (3.25). Nevertheless, the action is not fully invariant and requires a quartic term.

To demonstrate the invariance of (2.7) in our case it is instructive to consider an example. From (2.7) we can easily deduce the presence of the term with no derivatives

$$V_{\varphi\varphi\varphi} = -g (\varphi - S d - Zc)^3 + h.c + \dots \quad (3.29)$$

where  $\varphi$ ,  $c$  and  $d$  are the scalar components of (3.12) from triplets with spin 0, 1, 2 respectively. The omitted terms are proportional to  $c^2, c^3$  and have a gauge variation which does not mix with the gauge variation of the term in (3.29), so they will not affect our arguments below. The gauge variation of this term with respect to the scalar gauge parameter  $\lambda$  results in  $O(g^2)$  terms <sup>§</sup>

$$\delta V_{\varphi\varphi\varphi} = +3g^2 (1 - |S|^2) [(Z Z_a - Z_b^2) + (Z_b Z - Z_a^2)] c \lambda (\varphi - S d - Zc)^2. \quad (3.30)$$

This variation cannot be cancelled against the gauge variation of any other cubic term since the latter contain derivatives. The vanishing of (3.30) after using  $Z_b = -Z_a$  requires  $|S|^2 = 1$ . We see therefore from (3.14) that invariance of the action requires the non-abelian part of the gauge transformation of the scalar  $\varphi$  to cancel that of the scalar  $d$  which is the  $s = 0$  field of the  $s = 2$  triplet. This suggests that the whole tower of HS fields is necessary since the gauge transformation of each "top-spin" component of each triplet will be canceled against the gauge transformation of the "bottom-component" of another triplet. Notice that just as we explained below (3.23) the condition (3.24) is imposed in order to cancel terms  $\sim |c\rangle|\lambda\rangle$  when one checks the closure of the algebra. It is easy to deduce that the full action is invariant under (2.8) with the vertex (3.2).

Let us make several comments:

- Dropping the cyclicity constraint does not seem to alter the conclusions. In this case we will have  $|S_{ii}|^2 = 1$ ,  $i = 1, 2, 3$ .
- Despite the algebra being trivial it seems the vertex cannot be obtained from the free Lagrangian via some field redefinition. In other words the vertex (3.2) is not trivial in cohomologies of the BRST charge (2.3):  $|V\rangle \neq Q|W\rangle$  for some  $|W\rangle$ . One can show that only terms diagonal in the Hilbert spaces  $i, j$  can be removed from the exponent of (3.2) via a specific field redefinition scheme [10]. Moreover, notice that in (3.29) the presence of both scalars  $\varphi$  and  $d$  is required in order to have gauge invariance. The nonabelian gauge transformation of the

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<sup>§</sup>We have assumed that the coefficients  $S_{ii}$  and  $Z_{ij}$  are in general complex

one cancels the nonabelian part of the other. Their abelian parts differ though: The scalar  $\varphi$  has trivial abelian gauge transformation while  $\delta d \sim \partial^\mu \lambda_\mu$  as required by the gauge invariance of the spin two triplet. If one tries to remove the vertex by a field redefinition from the free Lagrangian one will mix the free equations of the triplets, which as we know decouple in the free limit.

- The infinite tower of triplets is essential for the closure. The nonabelian part of the gauge transformation of each component of  $|\varphi\rangle$  is canceled against the same rank tensor component of  $|d\rangle$ . However, the two tensors belong though to different triplets.
- In order to deal with the quantum theory one needs to gauge fix the action. The usual partial gauge fixing condition is the Siegel gauge  $b_0|\phi\rangle = 0$ . This gauge eliminates auxiliary fields  $|c\rangle$  as one can see from (2.7). Then obviously the algebra on the constrained string field closes without any constraint on the parameter  $S$ . However, the off-shell closure of the algebra requires (3.24).

## 4 An exact cubic vertex for mixed symmetry fields

The case of arbitrary mixed symmetry fields is completely analogous to the construction in section 3 for totally symmetric fields. As in (3.2) we make the ansatz

$$V = \exp \left( \sum_{n=1}^{\infty} Y_{ij}^{(n)} l_{ij}^{+, (n)} + Z_{ij}^{(n)} \beta_{ij}^{+, (n)} \right) \times \quad (4.1)$$

$$\exp \left( \sum_{n,m=1}^{\infty} S_{ij}^{(nm)} \gamma_{ij}^{+, (nm)} + P_{ij}^{(nm)} M_{ij}^{+, (nm)} \right)$$

where in this case we are summing over  $n, m$  as well. We put the oscillator level indices in parentheses in order to distinguish them from the Hilbert space ones. The oscillator algebra takes the form

$$[\alpha_\mu^{(m), i}, \alpha_\nu^{+, (n), j}] = \delta^{mn} \delta^{ij} \eta_{\mu\nu}, \quad \{c^{+, (m), i}, b^{(n), j}\} = \{c^{(m), i}, b^{+, (n), j}\} = \delta^{mn} \delta^{ij}. \quad (4.2)$$

The BRST invariance with respect to (2.3) implies (3.5) and (3.6)

$$\sum_{i=1}^3 \sum_{r=0}^{\infty} c^{+, (r), i} (Y_{is}^{(r)} l_0^{is} - Z_{is}^{(r)} l_0^{ss}) |-\rangle_{123} = 0 \quad (4.3)$$

$$\sum_{i=1}^3 \sum_{r=0}^{\infty} \{ c^{+, (r), i} \left( \frac{1}{2} (P_{il}^{(rs)} l^{+, (s), li} + P_{li}^{(sr)} l^{+, (s), li}) - S_{ik}^{(rs)} l^{+, (s), kk} \right) - \right. \\ \left. b_0^i c^{+, (p), i} c^{+, (r), m} p S_{mi}^{(rp)} \} |-\rangle_{123} = 0 \quad (4.4)$$

where the summation over repeated indexes is assumed. Solving (4.3) we get

$$Z_{i,i+1}^{(r)} + Z_{i,i+2}^{(r)} = 0 \quad (4.5)$$

$$Y_{i,i+1}^{(r)} = Y_{ii}^{(r)} - Z_{ii}^{(r)} - \frac{1}{2}(Z_{i,i+1}^{(r)} - Z_{i,i+2}^{(r)}) \quad (4.6)$$

$$Y_{i,i+2}^{(r)} = Y_{ii}^{(r)} - Z_{ii}^{(r)} + \frac{1}{2}(Z_{i,i+1}^{(r)} - Z_{i,i+2}^{(r)}). \quad (4.7)$$

Equation (4.4) gives

$$\begin{aligned} S_{ij}^{(ps)} &= P_{ij}^{(ps)} = 0 & i \neq j \text{ or } p \neq s \\ P_{ii}^{(ss)} - S_{ii}^{(ss)} &= 0 & i = 1, 2, 3 \end{aligned} \quad (4.8)$$

where unlike the case of totally symmetric fields the second term of (4.4) is not identically zero before taking matrices  $S$  and  $P$  to be diagonal in  $p$  and  $s$ . We can once more choose a cyclic solution in the three Hilbert spaces as in (3.13) and in this way get an obvious generalization of (4.5).

The discussion of the closure of the algebra follows closely the lines of subsection 3.2. The condition for the closure is again the equation (3.17). In addition, in the case of both mixed symmetry and totally symmetric fields we have a diagonal  $S_{ij}^{(ps)}$ . It is straightforward to show that  $|\tilde{\Lambda}\rangle$  vanishes as in (3.18). The steps similar to (3.19) and (3.21) lead us to the equivalent of (3.22)

$$\begin{aligned} \langle V || \Phi_1 \rangle | \Lambda_3 \rangle \langle \Xi_3 || V \rangle &= \sum_{n=0}^{\infty} Z_a^{(r_{n+1})} \mathcal{A}_3 \langle \Xi_{r_1 \dots r_n}^{r_1 \dots r_n} | A \rangle T^{(r_1 \dots r_{n+1})} \\ &\times \left( \sum_{m=0}^{\infty} \prod_{r \in \mathcal{S}_m} (1 - |S^{(r)}|^2) \right) e^{(\sum_p S^{(p)} c_1^{+, (p)} b_1^{+, (p)})} |0\rangle_{123} \end{aligned} \quad (4.9)$$

where we have shown only the terms proportional to the gauge parameter  $|\Lambda_{i_1 \dots i_l}^{j_1 \dots j_{l+1}}\rangle$

$$\begin{aligned} \mathcal{A} &\sim -\langle 0_2 | \sum_{l=0}^{\infty} \left( \langle A | D_{i_1, \dots, i_l}^{(i_1, \dots, i_l)} \rangle_1 | \Lambda_{j_1, \dots, j_n}^{(j_1, \dots, j_{n+1})} \rangle_3 \right) \sum_{j_{n+1}} Z_b^{(j_{n+1})} T_1^{(i_1 \dots i_l)} T_3^{j_1 \dots j_n} + \\ &+ \langle 0_2 | c_0^2 \left( \langle A | C_{i_1, \dots, i_l}^{(i_1, \dots, i_{l+1})} \rangle_1 | \Lambda_{j_1, \dots, j_n}^{(j_1, \dots, j_{n+1})} \rangle_3 \right) \\ &\times \sum_{i_{l+1}, j_{n+1}} (-Z^{(i_{l+1})} Z_b^{(j_{n+1})} + Z_a^{(i_{l+1})} Z_a^{(j_{n+1})}) T_1^{i_1 \dots i_l} T_3^{j_1 \dots j_n} \end{aligned} \quad (4.10)$$

and the set  $\mathcal{S}_m$  is defined as the set of all partitions of  $\mathbb{Z}$  in subsets of  $m$  arbitrary integers. Obviously this is an infinite set. The indices  $(i_r, j_r)$  are assumed to take values in  $\mathbb{Z}$  and label the levels of the oscillators involved. Every term in the summations over  $l$  in (4.10) or  $n$  in (4.9) has an infinite number of terms. This comes about since there are infinite tensor states of rank  $l$  (or rank  $n$ ). These are labeled

from the infinite number of subsets in  $\mathbb{Z}$  which are made by  $l$  (or  $m$ ) random integers. Obviously, in tension-full string theory these tensors have different masses, while in the tensionless limit all of them become massless. The tensor  $T_i^{j_1 \dots j_n}$  is defined as

$$T_i^{j_1 \dots j_n} = \sum_{l=0}^{\infty} \sum_{perm(j_r)} \left( \prod_{r=1}^l S_i^{(j_r)} \right) (-1)^{P(j_1 \dots j_l)} (-1)^l \quad (4.11)$$

where  $P(j_1 \dots j_l)$  stands for the permutations of the indices  $(j_1 \dots j_l)$ . Then  $(-1)^{P(j_1 \dots j_l)}$  gives  $+1$  for any even permutation of the set  $(j_1 \dots j_l)$  and  $-1$  for an odd permutation. The tensor defined above effectively sums over all possible ways of saturating the ghosts of vertex (4.1) with a given state like (2.7) or (2.11). In (4.10) the indices in parentheses are assumed to be contracted and therefore summed over. In a state  $|\Lambda_{j_1, \dots, j_n}^{(j_1, \dots, j_{n+1})}\rangle$  it is implied that the the first  $n$ -upper indices are equal to the lower ones and only the last  $j_{n+1}$  can differ.

Our expressions (4.9) and (4.10) are divergent due to the infinite degeneracy of the tensionless string spectrum. Nevertheless, as in the case of the totally symmetric states, one can show that the first term in (4.10) can cancel among the four different contributions on the LHS of (3.17). The second term though can cancel only if

$$|S^{(r)}|^2 = 1, \quad \forall r \in \mathbb{Z}. \quad (4.12)$$

The condition above can be shown along the lines of section 3.2 to ensure invariance of the action for the case of mixed symmetry fields as well.

This completes our treatment of the the whole spectrum of the open bosonic string at the high energy limit. The mechanism of the closure of the algebra and consequent gauge invariance of the vertex is the same as that in the subsection 3.2. Again, in the Siegel gauge (if imposed) the algebra of gauge transformations with constrained parameters closes without any constraint on the coefficients  $S_{ii}^{(r)}$  due to non-trivial cancelation among the four surviving terms of the LHS of (3.17).

## 5 AdS space

In order to extend the discussion of the previous section to the case of an arbitrary dimensional *AdS* space let us recall some relevant facts about the triplet formulation on anti-de Sitter space [7]–[9] (see also [11]–[13]) We restrict ourselves to the case of totally symmetric fields on  $\mathcal{D}$ -dimensional AdS space-time i.e., to the case of the "triplet". The formulas given in section 2 apply to the case of AdS space as well (see [10] for details) , but now the ordinary partial derivative replaced by the operator

$$p_\mu = -i \left( \nabla_\mu + \omega_\mu^{ab} \alpha_a^+ \alpha_b \right) , \quad (5.1)$$

where  $\omega_\mu^{ab}$  is the spin connection of AdS and  $\nabla_\mu$  is the AdS covariant derivative. The AdS counterpart of the BRST charge (2.3) has the form

$$Q = c_0(l_0 + \frac{1}{L^2}(N^2 - 6N + 6 + \mathcal{D} - \frac{\mathcal{D}^2}{4} - 4M^+M + c^+b(4N - 6) + b^+c(4N - 6) + 12c^+bb^+c - 8c^+b^+M + 8M^+bc)) + c^+l + cl^+ - c^+cb_0 \quad (5.2)$$

where  $l_0$  is the AdS covariant d'Alembertian,  $L$  is the radius of the AdS space and

$$N = \alpha^{\mu+}\alpha_\mu + \frac{\mathcal{D}}{2}, \quad M = \frac{1}{2}\alpha^\mu\alpha_\mu. \quad (5.3)$$

Following the same strategy as in the case of flat space time one can try to make an AdS deformation of the flat space-time solution. However, one can find that the direct AdS deformation of the solution (3.2) does not exist. In other words one can not find appropriate "counterterms" proportional to  $\frac{1}{L^2}$  in the vertex (3.2) which would make it a solution of equation<sup>¶</sup> (2.9).

The simplest way to see the problem is as follows. First let us note [10] that the operator

$$\tilde{N} = \alpha^{\mu,i+}\alpha_\mu^i + b^{i,+}c^i + c^{i,+}b^i \quad (5.4)$$

commutes with the BRST charge (5.2). So it is sufficient to check the consistency of (2.9) to any given level (eigenvalue of  $\tilde{N}$ ). Expanding vertex (3.2) to the first level it is straightforward to show that BRST invariance of the vertex is maintained. Expanding to level two we get the following terms:

$$V \sim \frac{1}{2}Y_{ij}Y_{mn}l^{+,ij}l^{+,mn} + Y_{ij}Z_{mn}l^{+,ij}\beta^{+,mn} + \frac{1}{2}Z_{ij}Z_{mn}\beta^{+,ij}\beta^{+,mn} + P_{ij}M^{+,ij} + S^{ij}\gamma^{+,ij}. \quad (5.5)$$

The coefficients  $Z_{ij}$ ,  $Y_{ij}$ ,  $P_{ij}$  and  $S_{ij}$  are now general functions of the AdS radius  $L^2$ . They are assumed to have an  $\frac{1}{L^2}$  expansion with the zeroth order term given by (3.7) and (3.9). One can easily see that just as in the flat case the last term of (5.2) results in  $S_{ij}$  to be diagonal. Further on the  $\frac{1}{L^2}$  terms of  $\hat{l}_0$  result in terms like  $c^{+,i}\alpha^{+,i}p^j$ ,  $j \neq i$ . These terms can only be canceled if we set  $Y_{12} = Y_{13}$ . That implies  $Z_a = 0$  and all off-diagonal components of the vertex are zero. This is a trivial vertex. So there is no non-trivial solution for AdS of the form (3.2).

A possible heuristic explanation of this fact is that anti-de Sitter space is not a solution of bosonic string theory. One might try to generalize the procedure described above to the high energy limit of the open superstring. This would require an analogous nilpotent BRST charge for fermionic massless higher spin fields as well as for mixed symmetry fields, thus finding a Lagrangian description for the equations obtained in [21] on possibly  $AdS_{\mathcal{D}} \otimes S_{\mathcal{D}'}$  background along the lines of [7]–[9]. This

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<sup>¶</sup>It does not mean of course that this equation does not have a solution at all [20].

seems possible since the BRST charge (5.2) is nilpotent not only for the case of Anti – de Sitter space but for any space–time with a constant curvature as well. In this case an AdS deformation of the vertex (3.2) might exist. We leave this interesting question for future work.

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